Transmit antenna selection based 3-D constellation design for JM-SM and JM-GSM

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Abstract: In this work, we propose a reduced complexity antenna selection technique for spatial modulation (SM) based 3-dimensional (3D) constellation design. In terms of average bit error rate (BER), the proposed scheme enhances performance of a previously proposed jointly-mapped SM (JM-SM) constellation design scheme, in which one set of bits is used to jointly select a combination of transmit antennas and constellation points for data transmission over independent identically distributed (i.i.d.) Rayleigh fading channels. Furthermore, we extend the scheme and its analysis to generalized SM (GSM) modulation, in which more than one antenna is activated for each constellation point transmission. The simulation results of JM-SM and JM-GSM modulations have been compared together as well as with their conventional counterparts. Simulation results show the superiority of the proposed scheme’s BER over JM-SM and conventional SM.

Key words: Generalized spatial modulation, semi-orthogonal antenna selection, spatial modulation, transmit antenna selection, 3-D modulation.

1. Introduction

Spatial modulation (SM) [1–3] has been proposed as a low-complexity multiple-input multiple-output (MIMO) concept employing spatial domain as well as signal domain in order to transmit more information through a MIMO wireless channel. In [4–8], the design of this technique and its advantages over a conventional MIMO system have been explained. Given its advantages, including low-complexity, high energy efficiency, inter-antenna interference avoidance, and low complexity synchronization, varieties of SM such as generalized spatial modulation (GSM), enhanced spatial modulation (ESM) and index modulation (IM) have been proposed [3, 9, 10]. Interested reader is referred to [11–13] for more related works on performance of SM.

GSM, originally proposed by [14], uses a subset of antennas to transmit the same constellation point. Its advantages, including enhancing spectral efficiency, achieving spatial diversity and increasing reliability of the wireless channel, it is considered as a good candidate to be adopted in future wireless communication systems.

In conventional SM, using one RF chain, it is not possible to gain transmit antenna diversity. To address this problem, a number of solutions have been proposed in [5, 15–17]. One attractive solution among these methods is transmit antenna selection (TAS) that provides diversity and enhances performance in MIMO channels [17, 18]. In terms of capacity and bit error rate, there are a number of algorithms combining TAS and SM [19–23]. Regarding TAS for SM, schemes based on Euclidean Distance (ED) have drawn much more attention. [24–28] have proposed methods that rely on ED based transmit antenna selection (ED-TAS). In [29],
an improved QRD-TAS (QRD-Based ED-TAS) has been proposed, which has a lower complexity and enhanced
performance. [29, 30] are examples of reduced complexity of ED-TAS. For the first time, in [18], ED-TAS
based on frequency selective channels is discussed. [31, 32] have proposed closed expression of union bound of
symbol error rate probability by using antenna selection in antenna grouping. One of the limitations of SM is
the number of transmit antennas, which must be a power of two. [33, 34] and [35] have proposed techniques
to overcome this limitation and allow the transmitter to have arbitrary numbers of antennas. In [35], a jointly
mapped spatial modulation (JM-SM) has been investigated in order to circumvent this impediment and an
efficient 3-D constellation design having low-complexity has been proposed. In addition, all of the advantages
offered by conventional SM have been maintained.

The well-known semi-orthogonal user selection (SUS) [36] is an algorithm, which is used in multi-user
MIMO to select the best subset of users. In [37], a simple form of SUS (Simplified Semi-orthogonal User
Selection (S-SUS) has been proposed leading to almost the same performance as exhaustive search, yet with
lower complexity. In this paper, we use S-SUS algorithm to select the best transmit antennas for jointly
mapped-GSM.

In this paper, we introduce constellation design schemes for JM-SM and JM-GSM based on schemes
previously used for TAS and user selection. By driving BER for proposed schemes and comparison with other
existing techniques, it is shown that JM-SM and JM-GSM with TAS and S-SUS achieve lower BER.

The organization of this paper is as follows: In Section 2, we describe the system model. In Section 3,
proposed scheme is presented. Simulation results and comparison have been illustrated in Section 4. Finally,
Section 5 concludes the paper.

Notation: |. | represents the absolute value. Other notations used in this paper are the Frobenius norm and
Hermitian transpose illustrated by ||.| |_F and (.)^H, respectively.

2. SYSTEM MODEL

2.1. JM-SM

Figure 1 shows a JM-SM system with N_t transmit antennas and N_r receive antennas. Each of the n transmit
bits is mapped to a transmit vector x_q ∈ C^{N_t}, q = 1, 2, ..., 2^n, which is a column of constellation design
τ_SM ∈ C^{N_t×2^n} given in:

\[ x_q = \begin{pmatrix} s_1^1 & \ldots & s_{m_1}^1 & 0 & \ldots & 0 & 0 & \ldots & 0 \\ 0 & \ldots & 0 & s_1^2 & \ldots & s_{m_2}^2 & 0 & \ldots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & \ldots & 0 & 0 & \ldots & 0 & s_{N_t}^{N_t} & \ldots & s_{m_{m_{N_t}}}^{N_t} \end{pmatrix}, \tag{1} \]

x_q consists of two parts, formulated according to x_q = e_i s_j^i = [0, \ldots, s_j^i, \ldots, 0]^T, e_i is a
vector having i-th element as one (active antenna) and the remaining elements (N_t – 1 elements) zero. s_j^i
is one of the signal constellation points transmitted by i-th antenna. In this work we consider quadrature
amplitude modulation (QAM) and phase shift keying (PSK) modulation types, in which s_j has the general
form of s_j = \frac{2}{\sqrt{E_c}} (s_j^i + is_j^Q). E_c is the average energy of the two dimensional constellation, i = \sqrt{-1}, and s_j^i
and s_j^Q are inphase and quadrature parts of each symbol, respectively. Values taken by s_j^i and s_j^Q depend
on modulation type; for example in 4-QAM, E_c = 2, s_j^i and s_j^Q take \{±1\} values. In JM-SM, the sub-signal
constellation transmitted by $i$-th active antenna is $L = \{s^1_i, s^2_i, \ldots, s^m_i\}$, $(0 \leq m_i \leq M)$, where $\sum_{i=1}^{N_t} m_i = 2^n$ should be satisfied, which removes the limitation imposed by conventional SM, in which both $N_t$ and $M$ must be a power of two resulting in $N_t M = 2^n$.

JM-SM mapper and receiver work according to the following steps:

- By considering input bits, one of the columns of Eq. (1) is selected, which determines the signal to be transmitted and its corresponding antenna to be activated.
- The main duty of receiver is to estimate $x_q$ by having the knowledge of the constellation points given in Eq. (1).
- Finally, the estimated $x_q$ is demodulated to the given corresponding information bits.

We assume frequency-flat fading channel. The received JM-SM signal can be expressed as

$$y = \sqrt{p} H x_q + n$$  \hspace{1cm} (2)$$

where $H \in \mathbb{C}^{N_r \times N_t}$ is the MIMO channel matrix, $p$ is the Signal-to-Noise Ratio (SNR) and $n \in \mathbb{C}^{N_r \times 1}$ is the noise vector whose elements are zero mean complex Gaussian random variables with variance $\sigma^2 I_{N_r}$.

**Figure 1.** Block diagram of Jointly Mapped SM (JM-SM)

### 2.2. JM-GSM

In this scheme, where two or more transmit antennas are active at one instant in time, transmission/reception procedures with $N_t$ transmit antennas and $N_p$ from $N_r$ antennas active is as follows:

- consider $\tau_{GSM}$, which consists of $2^n$ from $\left(\binom{N_t}{N_p}\right) \times M$ possible vectors. For example for the case of $N_t = 5$, 4-QAM and $n = 2$, one of the possible vector sets is shown below:

$$\tau_{GSM} = \begin{pmatrix}
\frac{1}{\sqrt{2}} (1 + i) & 0 & \frac{1}{\sqrt{2}} (1 + i) & 0 \\
0 & \frac{1}{\sqrt{2}} (1 - i) & 0 & \frac{1}{\sqrt{2}} (1 + i) \\
\frac{1}{\sqrt{2}} (1 - i) & 0 & \frac{1}{\sqrt{2}} (1 - i) & 0 \\
\frac{1}{\sqrt{2}} (1 + i) & 0 & \frac{1}{\sqrt{2}} (1 - i) & 0
\end{pmatrix}$$  \hspace{1cm} (3)$$
by considering input bits, one of the columns of the matrix $\tau_{GSM}$ is selected and signals of selected vector are transmitted by their corresponding active antennas.

- Receiver needs to detect the transmitted vectors among all vectors of $\tau_{GSM}$.
- Detected transmit vectors are de-mapped to the corresponding information bits.

The input-output equation is given by:

$$y = \sqrt{\frac{p}{N_p}} H x_q + n$$

(4)

where all the parameters are as the same as the ones defined in Eq. (2) and $N_p$ is the number of active antennas or RF-chains.

By assuming JM-SM($N_t, N_r, M, n$), there are $\binom{N_t M}{2^n}$ legitimate constellation points for JM-SM resulting in different BERs. Our goal is to find the best constellation in order to minimize BER by using TAS techniques.

3. Proposed Constellation Designs for JM-(G)SM

3.1. Proposed Constellation Design Using QRD-Based ED-TAS for JM-SM

In this section, we extend QRD-based ED antenna selection originally proposed for SM [29] to JM-SM.

Considering combination of $N_t$ transmit antennas and $M$ signal constellation points, our problem is to find $2^n$ of these combinations leading to the lowest possible BER. The combined transmit antenna-signal constellation is expressed as

$$\tilde{H} = [s_1 h_1, s_2 h_1, \ldots, s_M h_1, \ldots, s_M h_{N_t}]$$

(5)

where $h_i$ is the $i$-th column of matrix $H$. The goal is to select a subset having $2^n$ columns from above constellation leading to the best performance. We have $U_{Ns} = \binom{N_t M}{2^n}$ possible subset:

$$U_1 = \{1, 2, \ldots, 2^n\},$$

$$U_2 = \{1, 2, \ldots, 2^n - 1, 2^n + 1\},$$

$$\vdots$$

$$U_{Ns} = \{N_t \times M - 2^n + 1, \ldots, N_t \times M\},$$

(6)

Each of them corresponds to a $\tau_{SM} \in \mathbb{C}^{N_t \times 2^n}$, and results in an average BER. To select one of these combinations, we need to obtain free distance (FD) corresponding to that constellation, $d_{min}$, of that subset. In [28], $d_{min}$, which is the minimum distance of the received signal constellation points, has been defined as

$$d_{min}(X_s) = \min_{x_i^{(s)}, x_j^{(s)} \in X_s, x_i^{(s)} \neq x_j^{(s)}} \|x_i^{(s)} - x_j^{(s)}\|_F$$

(7)
where $x_i^{(s)} - x_j^{(s)}$ represents the difference of two column vectors of Eq. (5). $X_s$ consists of constellation vectors from $\tilde{H}$ corresponding to one of the $U_{N_s}$ selections.

In our proposed scheme, we suggest that a constellation subset, index by $\hat{s}$, be selected such that

$$\hat{s} = \text{arg} \max_{s \in \{1, \ldots, N_s\}} d_{\text{min}}(X_s)$$

and which results in best BER performance for JM-SM. $d_{\text{min}}(X_s)$ is defined in a simple form as [25]

$$d_{\text{min}}(X_s) = \min\{d_{\text{signal}}^{\text{joint}}, d_{\text{spatial}}^{\text{joint}}, d_{\text{joint}}^{\text{joint}}\};$$

where

$$d_{\text{signal}}^{\text{joint}} = \min_{i=1, \ldots, 2^n} \|\text{h}_s(i)\|_F^2 \min_{s_a \neq s_b \in S_M} |s_a - s_b|^2 = d_{\text{min}}^{\text{APM}} \|\text{h}_s(i)\|_F^2,$$

$$d_{\text{spatial}}^{\text{joint}} = \min_{i,j=1, \ldots, 2^n, i \neq j} \|\text{h}_s(i) - \text{h}_s(j)\|_F^2 \min_{s_l \in S_M} |s_l|^2 = d_{\text{min}}^{\text{Modulus}} \min_{i,j=1, \ldots, 2^n, i \neq j} \|\text{h}_s(i) - \text{h}_s(j)\|_F^2,$$

$$d_{\text{joint}}^{\text{joint}} = \min_{i,j=1, \ldots, 2^n, i \neq j} \|\text{h}_s(i)s_a - \text{h}_s(j)s_b\|_F^2$$

where $d_{\text{min}}^{\text{Modulus}}$ is the minimum squared modulus of transmit symbols and $\text{h}_s(i)$ is the $i$-th column of $H_s$. The main complexity of $d_{\text{min}}(H_s)$ is related to Eq. (12), so in order to reduce this complexity, QRD-based bound has been proposed in [29].

In [29], the upper bound of $d_{\text{min}}^{\text{joint}}$ has been approximated by

$$d_{\text{min}}^{\text{joint}} \geq d_{\text{min}}^{QRD-bound_F} = \min_{i,j=1, \ldots, 2^n, i \neq j} \{\|\tilde{R}_{QRQ, F}\|_F^2\} d_{\text{all}}^{\text{min}}$$

where

$$\|\tilde{R}_{QRQ, F}\|_F^2 = \max\{\|\tilde{R}_1\|_F^2, \|\tilde{R}_2\|_F^2\}$$

and

$$\|\tilde{R}_1\|_F^2 = \min\{\tilde{R}_{1,1}(\Pi_1), \tilde{R}_{2,2}(\Pi_1)\}$$

$$\|\tilde{R}_2\|_F^2 = \min\{\tilde{R}_{1,1}(\Pi_2), \tilde{R}_{2,2}(\Pi_2)\}$$

$$\tilde{R}_{1,1}(\Pi_1) = \sqrt{\|\text{h}_s(i)\|_F^2}$$

$$\tilde{R}_{2,2}(\Pi_1) = \sqrt{\|\text{h}_s(i)\|_F^2 + \|\text{h}_s(j)\|_F^2 - 2\Re\{\text{h}_s(i)\text{h}_s(j)^H\}}$$

$$\tilde{R}_{1,1}(\Pi_2) = \sqrt{\|\text{h}_s(j)\|_F^2}$$
As shown in Eqs. (17)-(20), complexity of proposed scheme compared to that of exhaustive searched based constellation design has been substantially reduced. In addition, the performance of the above scheme is very close to exhaustive-search-based, which has optimal performance with high complexity. By calculating above equations, we can find the BER-minimizing constellation for SM and JM-SM schemes.

3.2. Proposed SUS-based Constellation Design for JM-GSM

In this subsection we proposed a low complexity constellation design scheme based on SUS, orginally proposed for user selection in multi-user downlink systems. In JM-GSM, the optimal constellation is found via exhaustive search over all possible vectors, which imposes extremely high complexity when \( N_t \) is large. Our proposed scheme for vector (constellation point) selection is to build \( \tau_{GSM} \) constellation based on S-SUS \[37\]. The steps of proposed scheme are presented as follows:

Proposed S-SUS based Constellation Design for JM-GSM:

1) Initial \( \Omega_1 = \{1, 2, ..., N_t\} \), \( S = \phi \), \( j = 1 \), \( \gamma_{1,k} = \|\tilde{h}_k\|^2 \)
2) \( j \)-th transmit antenna \( J_j = \text{arg}_{k \in \Omega_j} \max \gamma_{j,k} \), \( S = S \cup \{J_j\} \)
3) \( a_j = \frac{i}{\sqrt{\gamma_{j,j}}} (\tilde{h}_{J_j} - \sum_{i=1}^{j-1} \tilde{h}_{J_i} a_{J_i}^H a_j) \)
4) If size of \( S \) is smaller than \( N_r \): \( \Omega_{j+1} = \{k \in \Omega_j, k \neq J_j, \|\tilde{h}_{J_j} a_j^H a_j\| < \alpha \|\tilde{h}_k\|\} \) for small positive constant \( \alpha \) for each \( k \in \Omega_{j+1} \), calculate \( \Omega_{j+1,k} = \Omega_{j,k} - \|\tilde{h}_{J_j} a_j^H a_j\| \)
   \( j = j + 1 \) and go to step 2
   Else Quite

Furthermore, \( \tilde{h}_k \in \mathbb{C}^{N_r \times 1} \) is the \( k \)-th column of Eq. (5), where is multi-user scenario corresponds to the channel of \( k \)-th user. However, here \( k \)-th column of (5) determines which transmit antennas are to be activated and which symbol is to be transmitted on those antennas. \( 0 < \alpha \leq 1 \) is a tuning parameter and has been set according to guidelines given in \[36\].

4. Simulation Result

In this section, we provide a BER comparison between our proposed TAS and SUS based constellation design schemes for JM-SM and JM-GSM with random antenna selection and conventional SM and GSM with the same number of transmit antennas. Simulation software used to obtain all results of this section is MATLAB.

Figure 2 and Figure 3 show BER performance of different constellation design schemes with 4-PSK and 4-QAM modulation types, respectively. In Figure 2, we consider a JM-SM (4,4,4-PSK,3), which is \( 4 \times 4 \) SM with a constellation matrix consisting of \( 2^3 \) columns. For further comparison, we consider the proposed scheme for SM with 4 transmit antennas and 3 bit per channel use (bpcu) and compare it with JM-SM with random antenna selection based constellation design with the same bpcu. In Figure 3, we investigate the same scenario 4-QAM modulation.
Figure 2. BER performance of SM with \( N_t = 4, 2, N_r = 4 \) antennas and JM-SM with \( N_t = 4, N_r = 4, 4\)-PSK modulation. All schemes in this plot transmit the same number of bits per channel use, i.e., 3 bits/channel use (bpcu).

Figure 3. BER performance SM with \( N_t = 4, 2, N_r = 4 \) antennas and JM-SM with \( N_t = 4, N_r = 4, 4\)-QAM modulation. All schemes in this plot transmit the same number of bits per channel use, i.e., 3 bits/channel use (bpcu).

Figure 4 shows BER performance of the proposed JM-GSM using 2 RF-chains, 4 transmit antennas, 4 receive antennas and 4-PSK modulation. In this figure, performance of JM-GSM with the random vector selection and conventional GSM, all with equal bpcu, have been compared. Figure 5 considers the same scenarios as Figure 5, with 4-QAM modulation.

In Figure 6, we made a comparison between the proposed JM-SM and JM-GSM for 2 and 3 bpcu. JM-SM with 2 bpcu achieves better performance as expected.

Figure 7 compares complexity of proposed transmit antennas selection scheme based constellation design.
Figure 4. The BER performance of different GSM and JM-GSM schemes with 2 RF-chains, $N_t = 4$, $N_r = 4$, 4PSK modulation and 3 bpcu.

Figure 5. The BER performance of different GSM and JM-GSM schemes with 2 RF-chains, $N_t = 4$ and $N_r = 4$, 4-QAM modulation and 3 bpcu.

where adopting S-SUS and SUS algorithms, and it shows the amount of complexity reduction of S-SUS compared to SUS. In terms of complexity, an upper bound in terms of floating point operations per second (flop) counts for TAS and S-SUS based schemes is $\varphi \leq O(N_t N_r^2)$ [37], which is lower than SUS in complexity order $O(N_t N_r^3)$ [36], where for complex number arithmetic, addition and subtraction are counted as 2 flops; multiplication and division are counted as 6 flops.
**Figure 6.** BER performance of JM-SM and JM-GSM with 2 RF-chains, 2 and 3 bpcu achieved by considering $N_t = 4$, $N_r = 4$ and 4-PSK modulation.

**Figure 7.** Complexity of different algorithms for different number of transmit antennas and $N_r = 4$.

### 5. CONCLUSION

This paper presented two 3-D constellation design schemes for JM-SM and JM-GSM. 3-D SM constellations circumvent impediment of conventional SM in terms of restricting the number of transmit antennas, which should be a power of two, while maintaining all its advantages. Optimum 3-D constellation design, especially for scenarios with large numbers of antennas and/or constellation size, is very complex and ultimately untraceable. Our proposed schemes are based on a TAS algorithm conceived for SM systems, low complexity QRD-Based antenna selection, and S-SUS based user selection schemes, which have been applied to JM-SM and JM-GSM. BER comparisons between each proposed method and its corresponding conventional one shows the effectiveness of the proposed schemes in terms of lowering BER. Finally, we showed the complexity of the proposed scheme in terms of flops count.

### References


